USN | | | | | | | | | | **10MAT41**

Fourth Semester B.E. Degree Examination, Dec.2018/Jan.2019
Engineering Mathematics - IV

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Using Taylor series method, solve $\frac{dy}{dx} = x^2 + y^2, y(0) = 1$ at the point x = 0.2, 0.3 consider up to 4^{th} degree term. **(06 Marks)**

b. Using Runge Kutta method of order 4, solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ with y(0) = 1 at x = 0.2, 0.4 by taking step length h0.2. **(07 Marks)**

c. Given $\frac{dy}{dx} = \frac{1}{2}xy$, y(0) = 1, y(0.1) =1.0025, y(0.2)=1.0101, y(0.3)=1.0228. Compute y at x = 0.4 by Adams – Bash forth predictor – corrector method use corrector formula twice. **(07 Marks)**

2 a. Evaluate y and z at x = 0.1 from the Picard's second approximation to the solution of the following system of equations given by y = 2 and z = 1 at x = 0 initially $\frac{dy}{dx} = x + z$ $\frac{dz}{dx} = x - y^2$. **(06 Marks)**

b. Given $y'' = x^3(y + y')$ with the initial condition y(0) = 1 y′(0) = 0.5 compute y(0.1) by taking h = 0.1 and using 4^{th} order Runge Kutta method. **(07 Marks)**

c. Applying Milne's method compute y(0.4) Given that y satisfies the equation $\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} - 6y = 0$ and y and y′ are governed by the following values
$y(0) = 1,\ y(0.1) = 1.03995,\ y(0.2) = 1.138036$
$y(0.3) = 1.29865,\ y'(0) = 0.1,\ y'(0.1) = 0.6955$
$y'(0.2) = 1.258,\ y'(0.3) = 1.873$. **(07 Marks)**

3 a. Derive Cauchy Riemann Equation in Cartesian form. **(06 Marks)**

b. Prove that for every analytic function f(z) = u + iv the two families of curves $u(x,y) = C_1$ and $v(x,y) = C_2$ form an orthogonal system. **(07 Marks)**

c. If $u - v = (x - y)(x^2 + 4xy + y^2)$ and f(z) = u + iv is analytic function of z = x + iy find f(z) interms of f(z). **(07 Marks)**

4 a. Find the bilinear transformation that maps the points z = 0, i, ∞ onto the points w = 1, −i, −1 respectively, find the invariant points. **(06 Marks)**

b. Discuss the transformation $w = e^z$. **(07 Marks)**

c. Evaluate $\int_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)} dz$, where c is the circle |z| = 3. **(07 Marks)**

PART – B

5 a. Starting from Laplace differential equation. Obtain Bessel's differential equation as $xy'' + xy' + (x^2 - n^2)y = 0$ **(08 Marks)**

b. If $x^3 + 2x^2 - x + 1 = a\,P_0(x) + b\,P_1(x) + c\,P_2(x) + d\,P_3(x)$ find the value of a, b, c, d. **(06 Marks)**

c. Derive Rodrigue's formula $P_n(x) = \dfrac{1}{2^n n!}\dfrac{dy}{dx^n}(x^2-1)^n$ **(06 Marks)**

6 a. Define axioms of probability. Prove that,
$P(A \cup B \cup C) = P(A) + P(B) + P(C) + P(A \cap B \cap C) - P(A \cap B) - P(B \cap C) - P(C \cap A)$ **(06 Marks)**

b. A solar water heater manufactured by a company consists of two parts the heating panel and the insulated tank. It is found that 6% of the heaters produced by the company have defective heating panels and 8% have defective tank. Find the percentage of non defective heaters produced by the company. **(07 Marks)**

c. A box contains 500 IC chips of which 100 are manufactured by company X and the rest by company Y. It is estimated that 10% of the chips made by company X and 5% made by company Y are defective. If a randomly selected chip is found to be defective find the probability that it came from company X. **(07 Marks)**

7 a. A random variables X takes the values −3, −1, 2 and 5 with respective probabilities $\dfrac{2k-3}{10}, \dfrac{k-2}{10}, \dfrac{k-1}{10}, \dfrac{k+1}{10}$. Find the value of k and i) $p(-3 < x < 4)$ ii) $p(x \le 2)$. **(06 Marks)**

b. Find the mean and variance of binomial distribution. **(07 Marks)**

c. In an examination 7% of students scores less than 35% marks and 89% of students score less than 60% marks. Find the mean and standard deviation of the marks are normally distribute, it is given that $P(0 < z < 1.2263) = 0.39$ and $P(0 < z < 1.4757) = 0.43$. **(07 Marks)**

8 a. Explain the following terms :
i) Null hypothesis
ii) Type I and Type II error
iii) Confidence limits. **(06 Marks)**

b. A coin is tossed 1000 times and it turn up head 540 times decide on the hypothesis that the coin is unbiased. **(07 Marks)**

c. A certain stimulus administered to each of the 12 patients resulted is the following change is blood pressure 5, 2, 8, −1, 3, 0, 6, −2, 1, 5, 0, 4 can it be calculated that the stimulus will increase the blood pressure ($t_{0.05}$ for 11 df 2.201.) **(07 Marks)**

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